Regionalized Regression Equations for Estimating Low-Flow Characteristics for Selected Louisiana Streams

By
Lucille S. Wright and Paul A. Ensminger

U.S. GEOLOGICAL SURVEY

Published by the
LOUISIANA DEPARTMENT OF TRANSPORTATION AND DEVELOPMENT
Baton Rouge, Louisiana

2004
STATE OF LOUISIANA
KATHLEEN BABINEAUX BLANCO, Governor

JOHNNY B. BRADBERRY, Secretary
DEPARTMENT OF TRANSPORTATION AND DEVELOPMENT

Edmond J. Preau, Jr., Assistant Secretary
OFFICE OF PUBLIC WORKS AND INTERMODAL

Zahir "Bo" Bolourchi, Chief
PUBLIC WORKS AND WATER RESOURCES DIVISION

Cooperative project with the
U.S. DEPARTMENT OF THE INTERIOR
GALE A. NORTON, Secretary

U.S. GEOLOGICAL SURVEY
Charles G. Groat, Director

Any use of trade, product, or firm names in this report is for descriptive purposes only and does not imply endorsement by the U.S. Government or the Louisiana Department of Transportation and Development.

For additional information contact:

Zahir “Bo” Bolourchi, P.E.
Chief, Public Works
and Water Resources Division
Louisiana Department of
Transportation and Development
P.O. Box 94245
Baton Rouge, LA 70804-9245
E-mail: BoBolourchi@dotd.louisiana.gov
Fax: (225) 274-4322
Telephone: (225) 274-4171
Home Page: http://www.dotd.louisiana.gov/intermodal/division/

Charles R. Demas
District Chief
U.S. Geological Survey
3535 S. Sherwood Forest Blvd., Suite 120
Baton Rouge, LA 70816-2255
E-mail: dc_la@usgs.gov
Fax: (225) 298-5490
Telephone: (225) 298-5481
Home Page: http://la.water.usgs.gov
CONTENTS

Abstract ................................................................. 1
Introduction .............................................................. 1
  Purpose and Scope ................................................... 2
  Previous Studies ...................................................... 2
  Acknowledgments ..................................................... 2
Description of Study Area ............................................. 3
  Physiographic Divisions ............................................ 3
  Hydrographic Regions .............................................. 3
Low-Flow Data ......................................................... 7
Description of Data Base .............................................. 7
Development of Regionalized Regression Equations ................. 11
  Logistic Regression .................................................. 11
  Weighted Least Squares Regression .............................. 13
  Final Equations ..................................................... 13
  Interactive FORTRAN Software Program ....................... 15
Limitations and Uncertainty .......................................... 16
Example Application .................................................. 18
Summary ................................................................. 19
Selected References ................................................... 19

APPENDIX

FORTRAN Code for Computation of Regional Low-Flow Values for Selected Louisiana Streams ........................................... 23

COMPACT DISC

[In pocket]

Regionalized Regression Equations for Computation of Low-Flow Values for Selected Louisiana Streams ........................................... 23

FIGURES

1-3. Maps showing:
  1. Mean annual precipitation normals for Louisiana for the base period, 1971-2000 ... 4
  2. Physiographic divisions and streams in Louisiana ......................... 5
  3. Low-flow hydrographic regions in Louisiana and locations of stations used in the regression analysis ........................................... 6
TABLES
1. Low-flow, basin, and climatic characteristics of stations included in the regression analysis. ................................................................. 8
2. Summary of regression equations and fit diagnostics. ....................... 15
3. Ranges of values for basin characteristics used to develop the regression equations. . . 16
4. Values needed to calculate prediction intervals ................................. 17

CONVERSION FACTORS AND DATUMS

<table>
<thead>
<tr>
<th>Multiply By</th>
<th>To obtain</th>
</tr>
</thead>
<tbody>
<tr>
<td>inch (in.)</td>
<td>25.4</td>
</tr>
<tr>
<td>foot (ft)</td>
<td>0.3048</td>
</tr>
<tr>
<td>foot per mile (ft/mi)</td>
<td>0.1894</td>
</tr>
<tr>
<td>cubic foot per second (ft³/s)</td>
<td>0.02832</td>
</tr>
<tr>
<td>square mile (mi²)</td>
<td>2.590</td>
</tr>
</tbody>
</table>

Temperature in degrees Fahrenheit (°F) can be converted to degrees Celsius (°C) as follows: °C = (°F - 32)/1.8.

Vertical coordinate information is referenced to the National Geodetic Vertical Datum of 1929 (NGVD 29)—a geodetic datum derived from a general adjustment of the first-order level nets of both the United States and Canada.

Horizontal coordinate information is referenced to the North American Datum of 1983.
REGIONALIZED REGRESSION EQUATIONS FOR ESTIMATING LOW-FLOW CHARACTERISTICS FOR SELECTED LOUISIANA STREAMS

By Lucille S. Wright and Paul A. Ensminger

ABSTRACT

This report presents equations for estimating the 7-day, 2-year (7Q2) and 7-day, 10-year (7Q10) low-flow characteristics for ungaged sites on selected Louisiana streams. Logistic and weighted least squares regressions were used to develop equations that related the magnitude of a low-flow characteristic from 50 continuous-record streamflow-gaging stations and 106 partial-record stations to significant basin and climatic characteristics. Only those stations which had no appreciable effects from regulation, had 10 or more years of streamflow record (10 or more base-flow measurements for partial-record stations), and had drainage areas less than 525 square miles were included in the analysis. Significant basin and climatic characteristics were drainage area, main channel slope, and mean annual precipitation normal.

Stations were partitioned into groups based on their locations within four previously defined hydrographic regions. Separate equations were obtained for each low-flow characteristic (7Q2 and 7Q10) within each region analyzed. An explanation of the limitations and an evaluation of the accuracy of the equations are included, as well as an example application.

INTRODUCTION

Louisiana has abundant surface-water resources. Most of the freshwater demands are met by surface-water sources, which provide substantial quantities of water for public supply, industrial use, power generation, and agricultural use (Sargent, 2002). The abundance of readily accessible surface water does not guarantee its availability and suitability for use. Increasing demands in terms of quantity and quality are placing greater stresses on the State’s water resources, especially during low-flow periods of late summer and early fall. Water availability is a major concern during these times (Ensminger and Wright, 2003).

Knowledge of the low-flow frequency characteristics of Louisiana streams is important in water-resource planning. A low-flow frequency characteristic (referred to in this report as a low-flow characteristic) is an estimate of the annual minimum average daily streamflow of a selected consecutive-day period for a given recurrence interval, in years. For example, the 7-day, 2-year (7Q2) and 7-day, 10-year (7Q10) low flows are defined as the annual minimum average daily streamflows for 7 consecutive days that have a recurrence interval of 2 and 10 years in length,
respectively. A recurrence interval is the average interval of time within which a specified consecutive-day (N-day) flow will be less than the indicated flow only once (Ensminger and Wright, 2003).

From data collected at continuous-record streamflow-gaging stations (referred to in this report as continuous-record stations) and partial-record stations operated by the U.S. Geological Survey (USGS), in cooperation with various Federal and State agencies, low-flow characteristics have been computed and are available to the public (Ensminger and Wright, 2003). Commonly, however, low-flow characteristics are needed at ungaged sites for which daily discharge records are not available. The USGS, in cooperation with the Louisiana Department of Transportation and Development (DOTD), conducted a study to develop equations for use in estimating low-flow characteristics for ungaged sites on selected streams.

**Purpose and Scope**

This report presents regionalized regression equations for estimating low-flow characteristics for ungaged sites on selected streams in Louisiana. Equations relating the magnitude of a low-flow characteristic to significant basin and climatic characteristics are reported for the 7Q\textsubscript{2} and 7Q\textsubscript{10} low flows. These equations are more precise than those developed earlier (Lee, 1985a), because of advances in regression analysis techniques and improved accuracy of the low-flow statistics due to longer periods of record. Additionally, the report includes an interactive FORTRAN program that was written to solve these equations. An explanation of the limitations for the use of the equations and an evaluation of their accuracy also are included, as well as an example application. The low-flow regression equations published in this report supersede all previously published low-flow regression equations for the study area.

**Previous Studies**

A previous USGS report on estimating low-flow characteristics at ungaged sites in Louisiana was published in 1985 and included streamflow data collected through 1981 (Lee, 1985a). In his report, Lee (1985a) developed a regionalization model for ungaged sites using ordinary least squares regression. Four regions were defined using information from geologic maps, soils maps, precipitation data, and low-flow data. Regression equations were developed that related the magnitude of a low-flow occurrence to drainage area, channel slope, and mean annual rainfall. Calculated estimates of the 7Q\textsubscript{2}, 7Q\textsubscript{10}, and 7-day, 20-year (7Q\textsubscript{20}) low flows for selected ungaged sites in the report yielded standard errors of regression from +/- 44 to +/- 61 percent. The basic low-flow statistics used in Lee’s (1985a) study were previously published in a separate report by Forbes (1980).

**Acknowledgments**

The authors thank Zahir “Bo” Bolourchi, Chief, Public Works and Water Resources Division, Louisiana Department of Transportation and Development, whose valuable suggestions enhanced the quality of this report. Appreciation also is extended to Gary D. Tasker, USGS retiree, for his expert advice, and the U.S. Army Corps of Engineers, who are partly responsible for the collection and distribution of data used in developing the low-flow statistics presented in this report.
DESCRIPTION OF STUDY AREA

The State has a humid-subtropical climate; average annual temperatures range from 63.1 °F in the northwest to 70 °F in the southeast (Southern Regional Climate Center, 2002). Mean annual precipitation (1971-2000) ranges from about 52 in. in the north to about 72 in. in the southeast (fig. 1) (Louisiana Office of State Climatology, 2002).

Physiographic Divisions

Louisiana lies within the Coastal Plain Physiographic Province, and includes parts of four physiographic divisions: (1) Pine Hills, (2) Prairies, (3) Coastal Marshes, and (4) Alluvial Plains (Fenneman, 1938) (fig. 2). Parts of north-central, western, and southeastern Louisiana are in the Pine Hills. The terrain is typically rolling hills, heavily forested with pine and some hardwood trees. A part of southwestern Louisiana is in the Prairies. The land-surface elevation in the Prairies ranges from about 20 to 30 ft above NGVD 29, and the area is generally treeless except along streams. Much of coastal Louisiana is in the Coastal Marshes. The terrain is extremely flat and subject to tidal flooding from the Gulf of Mexico. The flood plains adjacent to the Mississippi, Red, and Atchafalaya Rivers and east of the Ouachita River are in the Alluvial Plains. This terrain is flat. Interconnecting streams allow flow between the river basins (Lee, 1985b).

Hydrographic Regions

Stations were partitioned into groups based on their locations within previously defined hydrographic regions, described by Lee (1985a). In the study by Lee (1985a), the State was divided into four hydrographic regions based upon geologic, climatic, and low-flow data (fig. 3). In region 1, the $Q_{10}$ is zero for most of the streams. These streams are typically shallow, and channel beds are underlain by sediments including clay, silt, and sandy clay (Boniol, 1988). These relatively impermeable subsoils serve as a barrier between stream channels and underlying aquifer systems. As a result, connections to the aquifer systems are poor (Lee, 1985a).

Streams in region 2 have moderate to high, sustained, year-round flows. Stream channels in this region are incised into shallow sand and gravel deposits, allowing good connection to the aquifer systems which, in turn, provide plenty of water to these streams to maintain adequate, sustained flows during low-flow periods (Lee, 1985a).

In region 3, the sustained flow in streams is moderate to poor during low-flow periods of the year. Streams in this region flow across clay and silty sand and are weakly connected to aquifer systems (Lee, 1985a).

Many of the stream channels in region 4 have been modified by humans. Drainage divides are not easy to identify because many of the streams interconnect. Also, streams in the coastal zone are tidally affected; therefore, a stage-discharge relation is difficult to define (Lee, 1985a). Only stations in regions 1, 2, and 3 were used to develop regionalized regression equations described in this report. The data for region 4 (Mississippi River Delta and coastal zone) are not sufficient to define low-flow characteristics; therefore, this region was not included in the regression analysis.
Figure 1. Mean annual precipitation normals for Louisiana for the base period, 1971-2000. (Source: Louisiana Office of State Climatology, 2002)
Figure 2. Physiographic divisions and streams in Louisiana. (Source: Fenneman, 1938)
Figure 3. Low-flow hydrographic regions in Louisiana and locations of stations used in the regression analysis.
LOW-FLOW DATA

This study is based on the analysis of daily discharge records from 50 continuous-record stations and base-flow measurements from 106 partial-record stations. The locations of these stations are shown in figure 3. Continuous-record streamflow data include discharge records up to the 1999 water year; no partial-record streamflow data have been collected since 1975. Only those continuous-record stations having 10 or more years of streamflow record, no appreciable effects from regulation or other man-made modifications, and drainage areas less than 525 mi$^2$ were included in the analysis. Partial-record stations were limited to locations having 10 or more base-flow measurements and corresponding index stations that were free of regulation. Index stations are continuous-record stations located nearby that have 10 or more years of record and similar basin characteristics.

The streamflow-gaging station records and miscellaneous measurements are included in the USGS National Water Information System database. These records and the data from the partial-record stations are on file at the USGS office in Baton Rouge, Louisiana.

DESCRIPTION OF DATA BASE

Data used in the regression analysis consisted of low-flow, basin, and climatic characteristics. Initially, low-flow and basin characteristics were assembled in a Geographic Information System database. The 7Q$_2$ and 7Q$_{10}$ low-flow characteristics for the 50 continuous-record stations and the 106 partial-record stations were used as dependent variables. These low-flow characteristics were determined previously, using a mathematical procedure that fitted a Pearson type III distribution to the logarithms of the low-flow data (Ensminger and Wright, 2003). The regression analysis was performed using the low-flow characteristics and the explanatory (or independent) variables of the following basin and climatic characteristics:

1. Contributing drainage area (DA)—the total drainage area, measured in square miles;

2. Main channel slope (SL)—the slope of the streambed as determined from USGS topographic maps, measured between two points along the main channel that are at 10 percent and 85 percent of the channel length, measured in feet per mile; and

3. Mean annual precipitation normal (P)—the 1971-2000 average annual precipitation, measured in inches (fig. 1) (Southern Regional Climate Center, 2002; Louisiana Office of State Climatology, 2002).

Table 1 lists the stations and associated characteristics upon which the regression analysis is based. Included in table 1 are the values for 7Q$_2$ and 7Q$_{10}$ that have been determined for each station using the methods described by Ensminger and Wright (2003). Also listed are the map number, station identification number, station type, drainage area, main channel slope, mean annual precipitation normal, and hydrographic region for each station.
### Table 1. Low-flow, basin, and climatic characteristics of stations included in the regression analysis

[7Q2, 7-day, 2-year low flow; 7Q10, 7-day, 10-year low flow; PR, partial-record station; CR, continuous-record streamflow-gaging station]

<table>
<thead>
<tr>
<th>Station identification number</th>
<th>Drainage area (square miles)</th>
<th>Main channel slope (feet per mile)</th>
<th>Mean annual precipitation normal (inches)</th>
<th>Hydrographic region (see fig. 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2489300</td>
<td>42</td>
<td>32</td>
<td>PR</td>
<td>72.3</td>
</tr>
<tr>
<td>2 2489440</td>
<td>2.5</td>
<td>1.7</td>
<td>PR</td>
<td>14.2</td>
</tr>
<tr>
<td>3 2489470</td>
<td>4.3</td>
<td>2.9</td>
<td>PR</td>
<td>12.8</td>
</tr>
<tr>
<td>4 2490000</td>
<td>2.5</td>
<td>1.3</td>
<td>CR</td>
<td>12.1</td>
</tr>
<tr>
<td>5 2491200</td>
<td>23</td>
<td>18</td>
<td>PR</td>
<td>50.1</td>
</tr>
<tr>
<td>6 2491350</td>
<td>11</td>
<td>7.8</td>
<td>PR</td>
<td>42.2</td>
</tr>
<tr>
<td>7 2491700</td>
<td>15</td>
<td>10</td>
<td>PR</td>
<td>44.2</td>
</tr>
<tr>
<td>8 2491720</td>
<td>5.8</td>
<td>3.8</td>
<td>PR</td>
<td>9.44</td>
</tr>
<tr>
<td>9 2491890</td>
<td>.69</td>
<td>.37</td>
<td>PR</td>
<td>15</td>
</tr>
<tr>
<td>10 2491850</td>
<td>.65</td>
<td>.29</td>
<td>PR</td>
<td>8.75</td>
</tr>
<tr>
<td>11 2491870</td>
<td>1.1</td>
<td>.46</td>
<td>PR</td>
<td>8.97</td>
</tr>
<tr>
<td>12 2491900</td>
<td>.96</td>
<td>.29</td>
<td>PR</td>
<td>13.5</td>
</tr>
<tr>
<td>13 7344450</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>80.5</td>
</tr>
<tr>
<td>14 7347500</td>
<td>6.3</td>
<td>3</td>
<td>CR</td>
<td>364</td>
</tr>
<tr>
<td>15 7348800</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>66.9</td>
</tr>
<tr>
<td>16 7349100</td>
<td>1.5</td>
<td>.51</td>
<td>PR</td>
<td>43.6</td>
</tr>
<tr>
<td>17 7349200</td>
<td>.58</td>
<td>.11</td>
<td>PR</td>
<td>35.1</td>
</tr>
<tr>
<td>18 7349800</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>133</td>
</tr>
<tr>
<td>19 7350800</td>
<td>2.3</td>
<td>.44</td>
<td>PR</td>
<td>19</td>
</tr>
<tr>
<td>20 7351250</td>
<td>1.2</td>
<td>.38</td>
<td>PR</td>
<td>19.7</td>
</tr>
<tr>
<td>21 7351500</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>66</td>
</tr>
<tr>
<td>22 7351700</td>
<td>.45</td>
<td>.25</td>
<td>CR</td>
<td>19.5</td>
</tr>
<tr>
<td>23 7351900</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>35.1</td>
</tr>
<tr>
<td>24 7352000</td>
<td>9.5</td>
<td>4.4</td>
<td>CR</td>
<td>154</td>
</tr>
<tr>
<td>25 7352200</td>
<td>.25</td>
<td>.02</td>
<td>PR</td>
<td>38.6</td>
</tr>
<tr>
<td>26 7352400</td>
<td>3.9</td>
<td>2.1</td>
<td>PR</td>
<td>21.1</td>
</tr>
<tr>
<td>27 7352500</td>
<td>16</td>
<td>6.6</td>
<td>CR</td>
<td>423</td>
</tr>
<tr>
<td>28 7352600</td>
<td>1.5</td>
<td>.60</td>
<td>PR</td>
<td>21.5</td>
</tr>
<tr>
<td>29 7352700</td>
<td>2.9</td>
<td>1.3</td>
<td>PR</td>
<td>27.9</td>
</tr>
<tr>
<td>30 7352800</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>93.9</td>
</tr>
<tr>
<td>31 7354000</td>
<td>.96</td>
<td>.45</td>
<td>CR</td>
<td>21.4</td>
</tr>
<tr>
<td>32 7354100</td>
<td>6.5</td>
<td>2.8</td>
<td>PR</td>
<td>140</td>
</tr>
<tr>
<td>33 7354200</td>
<td>.61</td>
<td>.09</td>
<td>PR</td>
<td>51.1</td>
</tr>
<tr>
<td>34 7354500</td>
<td>.15</td>
<td>.04</td>
<td>CR</td>
<td>5.27</td>
</tr>
<tr>
<td>35 7354700</td>
<td>7.9</td>
<td>2.3</td>
<td>PR</td>
<td>360</td>
</tr>
<tr>
<td>36 7355000</td>
<td>7.0</td>
<td>5.5</td>
<td>CR</td>
<td>18</td>
</tr>
<tr>
<td>37 7355200</td>
<td>8.5</td>
<td>5.0</td>
<td>CR</td>
<td>26.4</td>
</tr>
<tr>
<td>38 7355360</td>
<td>.67</td>
<td>.30</td>
<td>PR</td>
<td>3.7</td>
</tr>
<tr>
<td>39 7364300</td>
<td>.51</td>
<td>0</td>
<td>CR</td>
<td>271</td>
</tr>
<tr>
<td>40 7364800</td>
<td>.16</td>
<td>0</td>
<td>PR</td>
<td>30</td>
</tr>
<tr>
<td>41 7364900</td>
<td>.04</td>
<td>0</td>
<td>PR</td>
<td>68.9</td>
</tr>
<tr>
<td>42 7365000</td>
<td>.29</td>
<td>0</td>
<td>CR</td>
<td>355</td>
</tr>
<tr>
<td>43 7365100</td>
<td>.32</td>
<td>0</td>
<td>PR</td>
<td>63.3</td>
</tr>
<tr>
<td>44 7365300</td>
<td>.04</td>
<td>0</td>
<td>PR</td>
<td>43.9</td>
</tr>
<tr>
<td>45 7365500</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>178</td>
</tr>
</tbody>
</table>
Table 1. Low-flow, basin, and climatic characteristics of stations included in the regression analysis—Continued
[7Q₂, 7-day, 2-year low flow; 7Q₁₀, 7-day, 10-year low flow; PR, partial-record station; CR, continuous-record streamflow-gaging station]

<table>
<thead>
<tr>
<th>Map number (see fig. 2)</th>
<th>Station identification number</th>
<th>7Q₂ (cubic feet per second)</th>
<th>7Q₁₀ (cubic feet per second)</th>
<th>Station type</th>
<th>Drainage area (square miles)</th>
<th>Main channel slope (feet per mile)</th>
<th>Mean annual precipitation normal (inches)</th>
<th>Hydrographic region (see fig. 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>46</td>
<td>7366000</td>
<td>2.0</td>
<td>0</td>
<td>CR</td>
<td>462</td>
<td>3.5</td>
<td>56</td>
<td>1</td>
</tr>
<tr>
<td>47</td>
<td>7366200</td>
<td>2.2</td>
<td>0.06</td>
<td>CR</td>
<td>208</td>
<td>3.7</td>
<td>56</td>
<td>1</td>
</tr>
<tr>
<td>48</td>
<td>7367600</td>
<td>.14</td>
<td>.02</td>
<td>PR</td>
<td>60</td>
<td>6.5</td>
<td>56</td>
<td>1</td>
</tr>
<tr>
<td>49</td>
<td>7369360</td>
<td>4.1</td>
<td>1.5</td>
<td>PR</td>
<td>64.7</td>
<td>7.1</td>
<td>60</td>
<td>3</td>
</tr>
<tr>
<td>50</td>
<td>7370200</td>
<td>.28</td>
<td>.03</td>
<td>PR</td>
<td>60</td>
<td>6.5</td>
<td>56</td>
<td>1</td>
</tr>
<tr>
<td>51</td>
<td>7370500</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>271</td>
<td>3</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>52</td>
<td>7370600</td>
<td>.47</td>
<td>.02</td>
<td>PR</td>
<td>127</td>
<td>4.4</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>53</td>
<td>7370700</td>
<td>.04</td>
<td>0</td>
<td>PR</td>
<td>58</td>
<td>7.1</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>54</td>
<td>7370750</td>
<td>.05</td>
<td>0</td>
<td>PR</td>
<td>47.6</td>
<td>8.1</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>55</td>
<td>7370820</td>
<td>.16</td>
<td>0</td>
<td>PR</td>
<td>117</td>
<td>8.9</td>
<td>59</td>
<td>1</td>
</tr>
<tr>
<td>56</td>
<td>7370980</td>
<td>.32</td>
<td>.02</td>
<td>PR</td>
<td>20</td>
<td>12.7</td>
<td>59</td>
<td>1</td>
</tr>
<tr>
<td>57</td>
<td>7371000</td>
<td>.01</td>
<td>0</td>
<td>CR</td>
<td>2.14</td>
<td>63.5</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>58</td>
<td>7371800</td>
<td>0</td>
<td>0</td>
<td>PR</td>
<td>81</td>
<td>7.2</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>59</td>
<td>7372100</td>
<td>0</td>
<td>0</td>
<td>PR</td>
<td>31</td>
<td>12.5</td>
<td>56</td>
<td>1</td>
</tr>
<tr>
<td>60</td>
<td>7372110</td>
<td>.29</td>
<td>.03</td>
<td>PR</td>
<td>24</td>
<td>10.8</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>61</td>
<td>7372500</td>
<td>.20</td>
<td>0</td>
<td>CR</td>
<td>92</td>
<td>7.1</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>62</td>
<td>7372600</td>
<td>5.9</td>
<td>2.6</td>
<td>PR</td>
<td>30</td>
<td>10.8</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>63</td>
<td>7372720</td>
<td>5.4</td>
<td>4.4</td>
<td>PR</td>
<td>29</td>
<td>11</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>64</td>
<td>7372900</td>
<td>4.4</td>
<td>2.9</td>
<td>PR</td>
<td>12</td>
<td>18.4</td>
<td>59</td>
<td>2</td>
</tr>
<tr>
<td>65</td>
<td>7373000</td>
<td>13</td>
<td>8.1</td>
<td>CR</td>
<td>51</td>
<td>11.3</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>66</td>
<td>7373050</td>
<td>1.5</td>
<td>.41</td>
<td>PR</td>
<td>6.5</td>
<td>20.6</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>67</td>
<td>7373133</td>
<td>2.4</td>
<td>.95</td>
<td>PR</td>
<td>13.6</td>
<td>17.2</td>
<td>58</td>
<td>3</td>
</tr>
<tr>
<td>68</td>
<td>7373200</td>
<td>1.4</td>
<td>.39</td>
<td>PR</td>
<td>32</td>
<td>5.6</td>
<td>59</td>
<td>3</td>
</tr>
<tr>
<td>69</td>
<td>7373250</td>
<td>13</td>
<td>9.1</td>
<td>PR</td>
<td>35.3</td>
<td>12.6</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>70</td>
<td>7373263</td>
<td>3.5</td>
<td>2.5</td>
<td>PR</td>
<td>17</td>
<td>14</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>71</td>
<td>7373264</td>
<td>.59</td>
<td>.24</td>
<td>PR</td>
<td>8.5</td>
<td>19.8</td>
<td>58</td>
<td>3</td>
</tr>
<tr>
<td>72</td>
<td>7373296</td>
<td>.72</td>
<td>.59</td>
<td>PR</td>
<td>4.32</td>
<td>28.7</td>
<td>63</td>
<td>2</td>
</tr>
<tr>
<td>73</td>
<td>7373300</td>
<td>19</td>
<td>13</td>
<td>PR</td>
<td>104</td>
<td>8.7</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>74</td>
<td>7373440</td>
<td>3.1</td>
<td>2.0</td>
<td>PR</td>
<td>11.1</td>
<td>21.5</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>75</td>
<td>7373450</td>
<td>22</td>
<td>16</td>
<td>PR</td>
<td>99.3</td>
<td>9.2</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>76</td>
<td>7373500</td>
<td>3.7</td>
<td>2.6</td>
<td>CR</td>
<td>35.3</td>
<td>11.8</td>
<td>63</td>
<td>2</td>
</tr>
<tr>
<td>77</td>
<td>7373570</td>
<td>4.5</td>
<td>3.4</td>
<td>PR</td>
<td>31.3</td>
<td>12.9</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>78</td>
<td>7373590</td>
<td>12</td>
<td>8.5</td>
<td>PR</td>
<td>66.6</td>
<td>9.7</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>79</td>
<td>7373610</td>
<td>2.2</td>
<td>1.5</td>
<td>PR</td>
<td>10.4</td>
<td>16.1</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>80</td>
<td>7373800</td>
<td>.03</td>
<td>0</td>
<td>PR</td>
<td>23.9</td>
<td>14.3</td>
<td>63</td>
<td>1</td>
</tr>
<tr>
<td>81</td>
<td>7374650</td>
<td>4.7</td>
<td>3.2</td>
<td>PR</td>
<td>16.4</td>
<td>12.3</td>
<td>65</td>
<td>2</td>
</tr>
<tr>
<td>82</td>
<td>7374700</td>
<td>27</td>
<td>21</td>
<td>PR</td>
<td>53.1</td>
<td>11.4</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>83</td>
<td>7375000</td>
<td>44</td>
<td>34</td>
<td>CR</td>
<td>95.5</td>
<td>7.1</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>84</td>
<td>7375150</td>
<td>30</td>
<td>16</td>
<td>PR</td>
<td>76.5</td>
<td>7.4</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>85</td>
<td>7375200</td>
<td>2.4</td>
<td>.93</td>
<td>PR</td>
<td>17.4</td>
<td>10.4</td>
<td>64</td>
<td>3</td>
</tr>
<tr>
<td>86</td>
<td>7375210</td>
<td>2.8</td>
<td>1.8</td>
<td>PR</td>
<td>16.8</td>
<td>11.6</td>
<td>64</td>
<td>3</td>
</tr>
<tr>
<td>87</td>
<td>7375220</td>
<td>.73</td>
<td>.07</td>
<td>PR</td>
<td>28.9</td>
<td>6.5</td>
<td>64</td>
<td>3</td>
</tr>
<tr>
<td>88</td>
<td>7375310</td>
<td>32</td>
<td>24</td>
<td>PR</td>
<td>59.6</td>
<td>8.2</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>89</td>
<td>7375400</td>
<td>7.7</td>
<td>5.9</td>
<td>PR</td>
<td>25.5</td>
<td>12.1</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>90</td>
<td>7375460</td>
<td>9.8</td>
<td>7.5</td>
<td>PR</td>
<td>24.4</td>
<td>11.9</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>Map number (see fig. 2)</td>
<td>Station identification number</td>
<td>(7Q_2) (cubic feet per second) (^1)</td>
<td>(7Q_{10}) (cubic feet per second) (^1)</td>
<td>Station type</td>
<td>Drainage area (square miles)</td>
<td>Main channel slope (feet per mile)</td>
<td>Mean annual precipitation normal (inches)</td>
<td>Hydrographic region (see fig. 3)</td>
</tr>
<tr>
<td>------------------------</td>
<td>-------------------------------</td>
<td>-------------------------------</td>
<td>-------------------------------</td>
<td>-------------</td>
<td>-----------------------------</td>
<td>---------------------------------</td>
<td>---------------------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>91</td>
<td>7375470</td>
<td>7.8</td>
<td>5.9</td>
<td>PR</td>
<td>27.9</td>
<td>8.9</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>92</td>
<td>7375800</td>
<td>38</td>
<td>32</td>
<td>PR</td>
<td>89.7</td>
<td>8.7</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>93</td>
<td>7375950</td>
<td>67</td>
<td>52</td>
<td>CR</td>
<td>136</td>
<td>7.6</td>
<td>68</td>
<td>2</td>
</tr>
<tr>
<td>94</td>
<td>7376000</td>
<td>12</td>
<td>9.1</td>
<td>CR</td>
<td>45</td>
<td>9.4</td>
<td>65</td>
<td>2</td>
</tr>
<tr>
<td>95</td>
<td>7375960</td>
<td>99</td>
<td>53</td>
<td>PR</td>
<td>220</td>
<td>7.2</td>
<td>68</td>
<td>2</td>
</tr>
<tr>
<td>96</td>
<td>7376000</td>
<td>97</td>
<td>78</td>
<td>CR</td>
<td>247</td>
<td>6.3</td>
<td>66</td>
<td>2</td>
</tr>
<tr>
<td>97</td>
<td>7376800</td>
<td>31</td>
<td>24</td>
<td>PR</td>
<td>123</td>
<td>7.2</td>
<td>67</td>
<td>2</td>
</tr>
<tr>
<td>98</td>
<td>7377050</td>
<td>18</td>
<td>15</td>
<td>PR</td>
<td>54.3</td>
<td>8.3</td>
<td>68</td>
<td>2</td>
</tr>
<tr>
<td>99</td>
<td>7377200</td>
<td>5.2</td>
<td>3.6</td>
<td>PR</td>
<td>27.3</td>
<td>12.5</td>
<td>68</td>
<td>2</td>
</tr>
<tr>
<td>100</td>
<td>7377400</td>
<td>29</td>
<td>19</td>
<td>PR</td>
<td>88</td>
<td>7.7</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>101</td>
<td>7377410</td>
<td>10</td>
<td>8.1</td>
<td>PR</td>
<td>25.7</td>
<td>11.5</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>102</td>
<td>7377500</td>
<td>45</td>
<td>36</td>
<td>CR</td>
<td>145</td>
<td>8.1</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>103</td>
<td>7377700</td>
<td>5.0</td>
<td>3.3</td>
<td>PR</td>
<td>42.4</td>
<td>9.7</td>
<td>64</td>
<td>2</td>
</tr>
<tr>
<td>104</td>
<td>7377782</td>
<td>0.16</td>
<td>0</td>
<td>CR</td>
<td>145</td>
<td>7.02</td>
<td>63</td>
<td>1</td>
</tr>
<tr>
<td>105</td>
<td>7381800</td>
<td>38</td>
<td>29</td>
<td>CR</td>
<td>68.3</td>
<td>8.4</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>106</td>
<td>7382000</td>
<td>80</td>
<td>53</td>
<td>CR</td>
<td>240</td>
<td>5.1</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>107</td>
<td>7382235</td>
<td>7.1</td>
<td>4.1</td>
<td>PR</td>
<td>10.4</td>
<td>16.9</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>108</td>
<td>7382238</td>
<td>9.0</td>
<td>6.0</td>
<td>PR</td>
<td>10.7</td>
<td>20.7</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>109</td>
<td>7382244</td>
<td>3.8</td>
<td>2.1</td>
<td>PR</td>
<td>5</td>
<td>28.5</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>110</td>
<td>7386000</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>37.1</td>
<td>2.2</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>111</td>
<td>7386500</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>19</td>
<td>1.8</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>112</td>
<td>8011500</td>
<td>0</td>
<td>0</td>
<td>CR</td>
<td>51.3</td>
<td>4.2</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>113</td>
<td>8012650</td>
<td>0.31</td>
<td>0.07</td>
<td>PR</td>
<td>18.7</td>
<td>12.3</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>114</td>
<td>8012700</td>
<td>1.1</td>
<td>0.17</td>
<td>PR</td>
<td>37.1</td>
<td>8.9</td>
<td>62</td>
<td>3</td>
</tr>
<tr>
<td>115</td>
<td>8013000</td>
<td>28</td>
<td>19</td>
<td>CR</td>
<td>499</td>
<td>3.8</td>
<td>61</td>
<td>3</td>
</tr>
<tr>
<td>116</td>
<td>8013200</td>
<td>2.1</td>
<td>1.1</td>
<td>PR</td>
<td>51.4</td>
<td>6.4</td>
<td>62</td>
<td>3</td>
</tr>
<tr>
<td>117</td>
<td>8013450</td>
<td>8.8</td>
<td>4.1</td>
<td>PR</td>
<td>79.7</td>
<td>3.9</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>118</td>
<td>8013650</td>
<td>2.1</td>
<td>0.53</td>
<td>PR</td>
<td>22</td>
<td>11</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>119</td>
<td>8013720</td>
<td>17</td>
<td>8.8</td>
<td>PR</td>
<td>128</td>
<td>6.8</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>120</td>
<td>8013900</td>
<td>20</td>
<td>13</td>
<td>PR</td>
<td>88.6</td>
<td>9.0</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>121</td>
<td>8013950</td>
<td>13</td>
<td>7.6</td>
<td>PR</td>
<td>34.4</td>
<td>10.9</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>122</td>
<td>8014200</td>
<td>14</td>
<td>9.8</td>
<td>CR</td>
<td>94.2</td>
<td>5.4</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>123</td>
<td>8014500</td>
<td>159</td>
<td>116</td>
<td>CR</td>
<td>510</td>
<td>5.8</td>
<td>61</td>
<td>3</td>
</tr>
<tr>
<td>124</td>
<td>8014550</td>
<td>3.7</td>
<td>2.1</td>
<td>PR</td>
<td>14.9</td>
<td>19.3</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>125</td>
<td>8014800</td>
<td>17</td>
<td>13</td>
<td>CR</td>
<td>120</td>
<td>7.7</td>
<td>60</td>
<td>2</td>
</tr>
<tr>
<td>126</td>
<td>8015600</td>
<td>0.07</td>
<td>0.12</td>
<td>PR</td>
<td>111</td>
<td>4.3</td>
<td>60</td>
<td>3</td>
</tr>
<tr>
<td>127</td>
<td>8015700</td>
<td>1.1</td>
<td>0.54</td>
<td>PR</td>
<td>23.1</td>
<td>8.6</td>
<td>60</td>
<td>3</td>
</tr>
<tr>
<td>128</td>
<td>8016200</td>
<td>0</td>
<td>0</td>
<td>PR</td>
<td>28.3</td>
<td>7.8</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>129</td>
<td>8016300</td>
<td>1.1</td>
<td>0.37</td>
<td>PR</td>
<td>76</td>
<td>5.8</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>130</td>
<td>8016400</td>
<td>1.6</td>
<td>0.35</td>
<td>CR</td>
<td>148</td>
<td>4.9</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>131</td>
<td>8016500</td>
<td>0.04</td>
<td>0</td>
<td>PR</td>
<td>34.9</td>
<td>6.8</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>132</td>
<td>8016600</td>
<td>0.04</td>
<td>0</td>
<td>CR</td>
<td>82.2</td>
<td>6.4</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>133</td>
<td>8016700</td>
<td>0.03</td>
<td>0</td>
<td>PR</td>
<td>45.6</td>
<td>5.7</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>134</td>
<td>8016800</td>
<td>0.01</td>
<td>0</td>
<td>CR</td>
<td>177</td>
<td>3.8</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>135</td>
<td>8016990</td>
<td>0</td>
<td>0</td>
<td>PR</td>
<td>15.3</td>
<td>6.8</td>
<td>58</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1. Low-flow, basin, and climatic characteristics of stations included in the regression analysis—Continued

\(7Q_2\), 7-day, 2-year low flow; \(7Q_{10}\), 7-day, 10-year low flow; PR, partial-record station; CR, continuous-record streamflow-gaging station.
DEVELOPMENT OF REGIONALIZED REGRESSION EQUATIONS

Two types of regression analyses, logistic and weighted least squares (WLS), were used in developing the final equations based upon data from 50 continuous-record stations and 106 partial-record stations in Louisiana. For region 1, a logistic regression was used first to estimate the probability of an annual minimum flow being zero. A WLS regression was then used to estimate the $7Q_2$ and $7Q_{10}$ low-flow characteristics. For regions 2 and 3, the WLS regression was sufficient. The basic concepts for each type of analysis are discussed in the following two sections.
Logistic Regression

A large percentage of streams in region 1 have a minimum flow of zero during a year. To accommodate the possibility of zero flow for an ungaged site within this region, a regional logistic regression equation was developed that related the probability of a zero minimum flow at the site to basin and climatic characteristics of the watershed (Tasker, 1989). The multiple logistic regression equation for nonzero flow is described as follows (Freund and Wilson, 1997):

\[ \pi(x) = \frac{e^{\mu(x)}}{1 + e^{\mu(x)}} , \]  \hspace{1cm} (1)

where

\[ \pi(x) = \text{the probability of nonzero flow, and} \]

\[ \mu(x) = \text{the logit transformation (Freund and Wilson, 1997) of the dependent variable (low-flow characteristic).} \]

The probability of zero flow is determined, conversely, by the equation

\[ 1 - \pi(x) = 1 - \left( \frac{e^{\mu(x)}}{1 + e^{\mu(x)}} \right) . \]  \hspace{1cm} (2)

Regression coefficients are determined using the following expression:

\[ \mu(x) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \ldots + \beta_nx_n , \]  \hspace{1cm} (3)

where

\[ \mu(x) = \text{the logit transformation of the dependent variable (low-flow characteristic);} \]

\[ \beta_0 \text{ to } \beta_n = \text{regression coefficients; and} \]

\[ x_1 \text{ to } x_n = \text{the independent variables (basin and climatic characteristics).} \]

Using sample data collected at the continuous-record and partial-record stations in the region, the unknown \( \beta \) values in equation 3 are estimated by maximizing the log-likelihood function
\[
L(\beta) = \sum_{i=1}^{n} \{y_i \ln[\pi(x)] + (1-y_i) \ln[1 - \pi(x)]\},
\]

(4)

in which \(y_i = 1\) if an observed annual minimum is nonzero in year \(i\) and \(y_i = 0\) if an observed annual minimum is zero and \(n\) is the total number of observations at all sites. PROC LOGISTIC (SAS Institute, 1999) was used to accomplish this.

**Weighted Least Squares Regression**

A WLS regression analysis was used to obtain the final equations. The WLS regression allotted each station in the analysis a different weight, assigned in accordance with the method described by Tasker (1980). The weights did not compensate for cross correlation between the streamflows of stations used in the regression analysis.

Dependent variables (low-flow characteristics) and independent variables (basin and climatic characteristics) were transformed to logarithmic units for regions 2 and 3 before regression analysis, and the equations were developed in logarithmic form. The equations relating the basin and climatic characteristics to the 7Q2 and 7Q10 low-flow characteristics, are as follows:

\[
\log_{10}y_i = \beta_0 + \beta_1 \log_{10}x_1 + \beta_2 \log_{10}x_2 + \ldots + \beta_n \log_{10}x_n + \epsilon_i ,
\]

(5)

or if retransformed,

\[
y_i = 10^{\beta_0}(x_1^{\beta_1})(x_2^{\beta_2})\ldots(x_n^{\beta_n})10^{\epsilon_i},
\]

(6)

where

\(y_i\) = the dependent variable (the 7Q2 or 7Q10 low-flow characteristic) for station \(i\);

\(x_1\) to \(x_n\) = the \(n\) independent variables (basin and climatic characteristics);

\(\beta_0\) to \(\beta_n\) = the \(n+1\) regression equation coefficients; and

\(\epsilon_i\) = the residual error for station \(i\).

Using an all-possible-subsets selection procedure using PROC REG in SAS initially, potential explanatory variables were identified based on \(R^2_{\text{adj}}\), a measure of the proportion of the total variation in dependent variable that is explained by the model, adjusted for the number of degrees of freedom; Mallow’s \(C_p\), an estimate of the standardized mean square error of prediction; and \(\text{mse}\), the mean square error (SAS Institute Inc., 1999). Model subsets then were investigated.
individually and final equations were determined based on a significance level of 0.05 for basin and climatic characteristics used in the analysis. An analysis of residuals also was performed with the Shapiro-Wilk normality test in PROC UNIVARIATE in SAS using a significance level of 0.05.

**Final Equations**

Separate regression equations were developed for regions 1, 2, and 3, and for each low-flow characteristic, $7Q_2$ and $7Q_{10}$. For an ungaged site in region 1, the first step is to compute the probability of the low-flow characteristic being zero, using the following equation:

$$1 - \pi(x) = 1 - \left( e^{\mu(x)} / \left( 1 + e^{\mu(x)} \right) \right),$$

where

$$\mu(x) = -1.1 + 0.00247DA + 0.0603SL + 0.163(P - 50).$$

The second step is to decide whether a regional equation should be used. If the result of the above equation indicates a probability greater than 0.5 for the $7Q_2$ estimation, the estimated low-flow characteristic is zero. If the probability is less than or equal to 0.5, the $7Q_2$ can be estimated from the following regional equation:

$$7Q_2 = 0.35 + 5.6E-5DA^2$$

Likewise, if the probability is greater than 0.1 for the $7Q_{10}$ estimation, the low-flow characteristic is estimated to be zero. If the probability is less than or equal to 0.1, the $7Q_{10}$ can be estimated from the following equation:

$$7Q_{10} = -7.1 + 0.0072DA + 5.5SL^{0.093}$$

For an ungaged site in region 2, the equations are as follows:

$$7Q_2 = 0.045DA^{0.83}(P-50)^{0.94}$$

$$7Q_{10} = 0.0015DA^{1.11}SL^{0.63}(P-50)^{1.17}$$

For an ungaged site in region 3, the equations are as follows:

$$7Q_2 = 8E-4DA^{1.30}SL^{1.46}$$

$$7Q_{10} = 1.6E-5DA^{1.58}SL^{2.31}$$

where

$$DA = \text{drainage area},$$
\( SL = \) main channel slope, and 

\( P = \) mean annual precipitation normal (fig. 1) (Louisiana Office of State Climatology, 2002; Southern Regional Climate Center, 2002).

Bias correction factors (BCF’s) were computed for regions 2 and 3 to account for the bias generated by detransforming the linear form of the equations into the exponential form presented in this report (Duan, 1983). To obtain the unbiased estimate, the exponential equations must be multiplied by

\[
\Sigma \exp^{\varepsilon_i} / N \tag{13}
\]

where

\( \varepsilon_i = \) the residual error at site \( i \), and

\( N = \) the number of stations used in the regression analysis.

The \( 7Q_2 \) BCF’s determined for this study were 1.03 and 1.04 for regions 2 and 3, respectively; the \( 7Q_{10} \) BCF’s were 1.03 and 1.07 for regions 2 and 3. A summary of the final equations is presented in table 2.

### Table 2. Summary of regression equations and fit diagnostics

<table>
<thead>
<tr>
<th>Equation number</th>
<th>Dependent variable</th>
<th>Equation</th>
<th>( \gamma )</th>
<th>( R^2 ) (adjusted)(^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>( 7Q_2 )</td>
<td>( 0.35 + 5.6E-5DA^2 )</td>
<td>0.31427</td>
<td>0.93</td>
</tr>
<tr>
<td>8</td>
<td>( 7Q_{10} )</td>
<td>(-7.1 + 0.0072DA + 5.5SL^{0.093})</td>
<td>0.32518</td>
<td>0.64</td>
</tr>
<tr>
<td>9</td>
<td>( 7Q_2 )</td>
<td>( 0.045DA^{0.83}(P-50)^{0.94} )</td>
<td>0.31696</td>
<td>0.78</td>
</tr>
<tr>
<td>10</td>
<td>( 7Q_{10} )</td>
<td>( 0.0015DA^{1.11}SL^{0.63}(P-50)^{1.17})</td>
<td>0.31705</td>
<td>0.78</td>
</tr>
<tr>
<td>11</td>
<td>( 7Q_2 )</td>
<td>( 8.0E-4DA^{1.30}SL^{1.46} )</td>
<td>0.21556</td>
<td>0.74</td>
</tr>
<tr>
<td>12</td>
<td>( 7Q_{10} )</td>
<td>( 1.6E-5DA^{1.58}SL^{2.31} )</td>
<td>0.33837</td>
<td>0.60</td>
</tr>
</tbody>
</table>
Interactive FORTRAN Software Program

An interactive FORTRAN program was written to solve the regression equations. The program is included on a compact disc and in printed form (appendix 1) at the back of this report. Values for the independent variables are input by the user. This program was modified from the program published within “Regionalization of Low-Flow Characteristics of Arkansas Streams” (Ludwig and Tasker, 1993). This program is for use within a virtual DOS environment, on a 386DX series or higher personal computer.

Although this program has been developed and used by the USGS, no warranty, expressed or implied, is made by the USGS or the United States Government as to the accuracy and functioning of the program and related program material nor shall the fact of distribution constitute any such warranty, and no responsibility is assumed by the USGS in connection therewith.

LIMITATIONS AND UNCERTAINTY

Equations in this report were designed to estimate the $7Q_2$ and $7Q_{10}$ low flows for ungaged sites free from appreciable effects of regulation in regions 1, 2, and 3 (fig. 3). Use of the equations should be limited to the ranges of values for drainage area, main channel slope, and mean annual precipitation normal used to develop the equations for sites in the appropriate region (table 3).

Table 3. Ranges of values for basin characteristics used to develop the regression equations
[NA, not applicable]

<table>
<thead>
<tr>
<th>Basin characteristic</th>
<th>Name in equations</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drainage area (square miles)</td>
<td>DA</td>
<td>5.27-360</td>
<td>2.08-423</td>
<td>3.7-510</td>
</tr>
<tr>
<td>Main channel slope (feet per mile)</td>
<td>SL</td>
<td>3.7-25.9</td>
<td>2.2-47.8</td>
<td>3.8-21.9</td>
</tr>
<tr>
<td>Mean annual precipitation index (inches)</td>
<td>P</td>
<td>52-60</td>
<td>52-72</td>
<td>NA</td>
</tr>
</tbody>
</table>

The accuracy of a regression equation can be evaluated by its coefficient of determination, residual analysis, and standard error of prediction. The coefficient of determination and model residuals are discussed in the section, “Development of Regionalized Regression Equations.” The model standard errors ($\gamma$) for the equations estimating $7Q_2$ and $7Q_{10}$ are given in table 2.
Another measure of uncertainty is the prediction interval of an estimate at an ungaged site. A 100 \((1-\alpha)\) prediction interval for the true low-flow characteristic, \(\theta_{7,\tau}\), at an ungaged site can be computed by

\[
\frac{1}{T}Q, \quad < \theta_{7,\tau}, \quad < (T)Q,
\]

where the value of \(T\) is computed as

\[
T = 10^{\frac{t(\alpha/2, n-p)S_i}{\gamma^2}}.
\]

For instances where bias correction is needed, \(Q\) is divided by the BCF which is discussed in the section, “Final Equations.” The variable \(t(\alpha/2, n-p)\) is the critical value from the students t-distribution at alpha level = 0.10 for a 90-percent prediction interval; \(n-p\) is the degrees of freedom where \(n\) is the number of stations used in the regression analysis and \(p\) is the number of independent variables used in the regression analysis, plus 1; and \(S_i\), the standard error of prediction for site \(i\), is computed from

\[
S_i = \left[\gamma^2 + x_i'Ux_i\right]^{0.5},
\]

where \(\gamma^2\) is the model error variance, \(x_i\) is a row vector of the basin characteristics for site \(i\), augmented by a 1 as the first element; \(U\) is the covariance matrix for the regression coefficients; and \(x_i'\) is the transpose of \(x_i\) (Ludwig and Tasker, 1993; Tasker and Driver, 1988). The applicable BCF’s and the values of \(t, \gamma^2,\) and \(U\) needed to determine prediction intervals for estimates obtained from the equations are presented in table 4.

**Table 4.** Values needed to calculate prediction intervals

<table>
<thead>
<tr>
<th>Equation number</th>
<th>Dependent variable</th>
<th>BCF</th>
<th>(t)</th>
<th>(\gamma^2)</th>
<th>(U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>(7Q_2)</td>
<td>-</td>
<td>1.78</td>
<td>0.09877</td>
<td>-3.07144E-7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.88380E-11</td>
</tr>
<tr>
<td>8</td>
<td>(7Q_{10})</td>
<td>-</td>
<td>1.80</td>
<td>0.10574</td>
<td>8.24294</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.0036180</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-6.47335</td>
</tr>
<tr>
<td>Region 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.0036180</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.53810E-6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0027768</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-6.47335</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0027768</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.09403</td>
</tr>
<tr>
<td>Region 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Values needed to calculate prediction intervals

[BCF, bias correction factor; t, critical value from the t-distribution; $\gamma^2$, model error variance; $U$, covariance matrix for regression coefficients; $7Q_2$, 7-day, 2-year low flow; $7Q_{10}$, 7-day, 10-year low flow]

<table>
<thead>
<tr>
<th>Region</th>
<th>$7Q_2$</th>
<th>$7Q_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1.03</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>0.10046</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.027385</td>
<td>-0.004469</td>
</tr>
<tr>
<td></td>
<td>-0.004469</td>
<td>0.003359</td>
</tr>
<tr>
<td></td>
<td>-0.004469</td>
<td>-0.00065343</td>
</tr>
<tr>
<td></td>
<td>-0.019153</td>
<td>-0.00065343</td>
</tr>
<tr>
<td></td>
<td>-0.019153</td>
<td>0.019556</td>
</tr>
<tr>
<td>10</td>
<td>1.03</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>0.10052</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.19575</td>
<td>-0.053660</td>
</tr>
<tr>
<td></td>
<td>-0.053660</td>
<td>0.018728</td>
</tr>
<tr>
<td></td>
<td>-0.053660</td>
<td>0.035635</td>
</tr>
<tr>
<td></td>
<td>-0.12078</td>
<td>0.035635</td>
</tr>
<tr>
<td></td>
<td>-0.12078</td>
<td>0.090642</td>
</tr>
<tr>
<td></td>
<td>0.01184</td>
<td>-0.011531</td>
</tr>
<tr>
<td></td>
<td>0.01184</td>
<td>-0.0026505</td>
</tr>
<tr>
<td></td>
<td>0.01184</td>
<td>0.032207</td>
</tr>
<tr>
<td>Region 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1.04</td>
<td>1.72</td>
</tr>
<tr>
<td></td>
<td>0.04647</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.63652</td>
<td>-0.15047</td>
</tr>
<tr>
<td></td>
<td>-0.15047</td>
<td>0.040292</td>
</tr>
<tr>
<td></td>
<td>-0.15047</td>
<td>0.089652</td>
</tr>
<tr>
<td></td>
<td>-0.40850</td>
<td>0.089652</td>
</tr>
<tr>
<td></td>
<td>-0.40850</td>
<td>0.27601</td>
</tr>
<tr>
<td>12</td>
<td>1.07</td>
<td>1.72</td>
</tr>
<tr>
<td></td>
<td>0.11449</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.38662</td>
<td>-0.32715</td>
</tr>
<tr>
<td></td>
<td>-0.32715</td>
<td>0.08746</td>
</tr>
<tr>
<td></td>
<td>-0.32715</td>
<td>0.19511</td>
</tr>
<tr>
<td></td>
<td>-0.89094</td>
<td>0.19511</td>
</tr>
<tr>
<td></td>
<td>-0.89094</td>
<td>0.60271</td>
</tr>
</tbody>
</table>
EXAMPLE APPLICATION

Low-flow characteristics, $7Q_2$ and $7Q_{10}$, for an ungaged site can be estimated by entering the drainage area, slope, and mean annual precipitation normal for the ungaged site into the equations developed from the regression analysis. An example is given below to show how the calculation can be done. Consider an ungaged site in region 1 with a drainage area of 355 mi$^2$, a slope of 4.7 ft/mi, and a mean annual precipitation normal of 56 in. The calculations for the $7Q_2$ and $7Q_{10}$ would be made as follows.

The first step is to calculate the probability of zero flow. Starting with equation 3:

$$
\mu(x) = -1.1 + 0.00247(355) + 0.0603(4.7) + 0.163(56-50) = 1.038
$$

and the probability of zero flow can be obtained from equation 2 as:

$$
1 - \pi(x) = 1 - \left(\frac{e^{(1.038)}}{1 + e^{(1.038)}}\right) = 0.26
$$

Because the probability is less than 0.5, the $7Q_2$ is estimated by equation 7:

$$
7Q_2 = 0.35 + 5.6E-5(355)^2 = 7.4
$$

To determine the 90-percent prediction interval, the value for the model variance, $\gamma^2$, can be obtained from table 4. The $x_i$ vector is $x_i = (1,355)$ and the covariance matrix, $U$, is obtained from table 4.

$$
U = \begin{bmatrix}
0.02595 & -3.07144E-7 \\
-3.07144E-7 & 1.88380E-11
\end{bmatrix}
$$

The standard error of prediction, $S_i$, is computed from equation 16:

$$
S_i = [\gamma^2 + x_iUx_i]^{0.5} = [0.09877 + 0.02573]^{0.5} = 0.353
$$

and $T$ is calculated from equation 15:

$$
T = 10^{(1.78*0.353)} = 4.24.
$$

The 90-percent prediction interval is estimated from equation 14 as:

$$
\frac{1}{T}(7.4) < 7Q_2 < 7.4(T) = 1.7 < 7Q_2 < 31.5
$$

Because the probability of zero flow is greater than 0.1, the $7Q_{10}$ is estimated to be zero.
SUMMARY

Regionalized regression equations were developed to estimate the 7-day, 2-year (7Q2) and 7-day, 10-year (7Q10) low-flow characteristics for ungaged sites on selected Louisiana streams. The equations were based on the analysis of daily discharge records from 50 continuous-record streamflow-gaging stations and base-flow measurements from 106 partial-record stations. Only those continuous-record stations having 10 or more years of streamflow record, no appreciable effects from regulation or other man-made modifications, and drainage areas less than 525 square miles were included in the analysis. Partial-record stations were limited to locations having 10 or more base-flow measurements and corresponding index stations that were free of regulation.

The equations were developed to relate low-flow characteristics to significant basin and climatic characteristics for stations analyzed. Basin and climatic characteristics used in the analysis were drainage area, main channel slope, and mean annual precipitation normal. The stations were partitioned into four previously defined hydrographic regions, and separate equations were developed for each low-flow characteristic (7Q2 and 7Q10) within each region analyzed. For region 1, a logistic regression was utilized first to estimate the probability of an annual minimum flow being zero, and then a weighted least squares regression was used to estimate the 7Q2 and 7Q10 low-flow characteristics. For regions 2 and 3, a weighted least squares regression was sufficient. The data for region 4 (Mississippi River Delta and coastal zone) are not sufficient to define low-flow characteristics; therefore, this region was not included in the regression analysis. An interactive FORTRAN program was written to solve the regression equations. Values for the independent variables are input by the user. The FORTRAN program is provided on a compact disc and in printed form at the back of this report.

Equations in this report were designed to estimate the 7Q2 and 7Q10 low flows for ungaged sites that are free from appreciable effects of regulation in regions 1, 2, and 3. Use of the equations should be limited to the ranges of values for drainage area, main channel slope, and mean annual precipitation normal used to develop the equations for sites in the appropriate region. An explanation of the limitations and an evaluation of the accuracy of the equations also are included, as well as an example application.

SELECTED REFERENCES


APPENDIX

FORTRAN Code for Computation of Regional Low-Flow Values for Selected Louisiana Streams
APPENDIX

FORTRAN Code for Computation of Regional Low-Flow Values for Selected Louisiana Streams

! Program regression analysis

character*32 site
character*80 hdng,temp,t11,t12,t13,t14,t15,t16
character*2 regnum
character*3 go
real*4 da, sl, pre, xlda, xslsl, xlp
integer*4 region, warn
common /c1/ da, sl, pre, xlda, xslsl, xlp, q72, q710, b72, &
b710, u72, u710, warn
!
!
t11(25:) = 'Regionalized Regression Analysis'
t12(14:) = 'of Low-Flow Characteristics on Selected Louisiana Streams'
t13(40:) = 'By'
t14(25:) = 'L.S. Wright and P.A. Ensminger'
t15(14:) = 'This program computes estimates of the 7-day 2-year and'
t16(14:) = '7-day 10-year flow values at ungaged sites in Louisiana.')'
6000 continue
  write(*,*)
  write(*,*)
  write(*,*)
  write(*,'(A/)') t11
  write(*,'(A/)') t12
  write(*,'(A/)') t13
  write(*,'(A/)') t14
  write(*,*)
  write(*,*)
  write(*,'(A/)') t15
  write(*,'(A/)') t16
  write(*,*)
  write(*,*)
  write(*,*)
  write(*,2000)
2000 format(' Enter site id, if any (q or Q to quit)')
  read(*,1010)site
  if (site .EQ. 'q' .OR. site .EQ. 'Q') then
    goto 200
  else if (site .EQ. '') then
    site = '<no site id given>'
  endif
1010 format(a32)
400 continue
write(*,2010)
2010 format( &
   ' Enter region number from figure #3 (1,2,3, or 99 to quit'))
read(*,*)region
if (region .EQ. 99 ) then
   goto 200
else if((region .LT. 1) .OR. &
   (region .GT. 3 .AND. region .LT. 99) .OR. &
   (region .GT. 99)) then
   write(*,*)
   write(*,*)
   write(*,'(A/)') ' Invalid region: Please enter 1, 2, or 3.'
   write(*,*)
   goto 400
endif
if(region.eq.1) then
   goto 411
endif
if(region.eq.2) then
   goto 3090
endif
if (region.eq.3) then
   goto 5029
endif
3090 write(*,4000)
4000 format(' Enter drainage area, in square miles')
read(*,*)da
if (da .LT. 2.08 .OR. da .GT. 423) then
   write(*,*)
   write(*,*) &
   ' Value out of range: Check applicable ranges in &
   &table 3, p.16 of the report.'
   write(*,*)
   goto 3090
end if
xlda=alog10(da)
4029  write(*,4030)
4030  format(' Enter slope, in feet per mile or ("99" if not applicable)')
read(*,*)sl
xsl=alog10(sl)
if (sl .EQ. 99) then
   sl=0
   goto 4041
elseif (sl .LT. 2.2 .OR. sl .GT. 47.8) then
   write(*,*)

write(*,*) &
' Value out of range: Check applicable ranges in &
&table 3, p.16 of the report.'
write(*,*)
goto 4029
end if
4041 write(*,4042)
4042 format(' Enter precipitation index, in inches')
read(*,*)pre
if (pre .LT. 52 .OR. pre .GT. 72) then
  write(*,*)
  write(*,*) &
  ' Value out of range: Check applicable ranges in &
  &table 3, p.16 of the report.'
  write(*,*)
goto 4041
end if
red=pre-50
xlp=alog10(red)
goto 2998
5029 write(*,5030)
5030 format(' Enter drainage area, in square miles')
read(*,*)da
if (da .LT. 3.7 .OR. da .GT. 510) then
  write(*,*)
  write(*,*) &
  ' Value out of range: Check applicable ranges in &
  &table 3, p.16 of the report.'
  write(*,*)
goto 5029
end if
xlda=alog10(da)
5039 write(*,5040)
5040 format(' Enter slope, in feet per mile')
read(*,*)sl
if (sl .LT. 3.8 .OR. sl .GT. 21.9) then
  write(*,*)
  write(*,*) &
  ' Value out of range: Check applicable ranges in &
  &table 3, p.16 of the report.'
  write(*,*)
goto 5039
end if
xlsl=alog10(sl)
goto 2998
411 write(*,2030)
2030 format(' Enter drainage area, in square miles')
read(*,*) da
  if (da .LT. 5.27 .OR. da .GT. 360) then
    write(*,*)
    write(*,*) &
      ' Value out of range: Check applicable ranges in &
      &table 3, p.16 of the report.'
    write(*,*)
    goto 411
  end if
end

2039  write(*,2040)
2040  format(' Enter slope, in feet per mile')
read(*,*) sl
  if (sl .LT. 3.7 .OR. sl .GT. 25.9) then
    write(*,*)
    write(*,*) &
      ' Value out of range: Check applicable ranges in &
      &table 3, p.16 of the report.'
    write(*,*)
    goto 2039
  end if
end

2041 write(*,2042)
2042 format(' Enter precipitation index, in inches')
read(*,*) pre
  if (pre .LT. 52 .OR. pre .GT. 60) then
    write(*,*)
    write(*,*) &
      ' Value out of range: Check applicable ranges in &
      &table 3, p.16 of the report.'
    write(*,*)
    goto 2041
  end if
end

! 2998 write(*,*)
  if(region.eq.1) call r1
  if(region.eq.2) call r2
  if(region.eq.3) call r3

!  write(*,*)
  write(*,*)
  write(*,(A/)) t11
  write(*,(A/)) t12
write(*,'(A/')) t13
write(*,'(A/')) t14
write(*,*)
write(*,*)
temp(1:27) = ' Estimate of low flows for '
temp(28:) = site
i = 1
if (site .EQ. '<no site id given>') then
  i = 19
else
  500 continue
  if (site(i:i) .NE. ' ') then
    i = i + 1
    goto 500
  endif
endif
write(regnum,'(I2)') region
temp(28+i:) = 'in region '
temp(38+i:) = regnum
hdng = ''
hdng((80-(39+i))/2:) = temp
write(*,'(A//)') hdng
write(*,*)
2999 format(' Flow Char.',t20,'Estimated',t35, &
  '90 Percent Prediction Interval',/,'Value',t35, &
  'Lower limit   Upper limit',/)
write(*,2999)
write(*,3000) q72, b72, u72
write(*,3001) q710, b710, u710
3000 format(1x,'Q(7,2)',t9, 3f15.1)
3001 format(1x,'Q(7,10)',t9, 3f15.1)
if(warn.eq.1)write(*,3002)
3002 format///,' WARNING Estimate beyond range of observed data')
!
!      stop
!      go = ''
6001 write(*,3003)
3003 format///,' Do you have another site? Enter (<y> = yes) or (<cr> = quit)'
read(*,3004) go
3004 format(a3)
if (go .eq. 'y' .or. go .eq. 'Y' .or. go .eq. 'yes' &
  .or. go .eq. 'YES') then
  goto 6000
elseif (go .eq. '' .or. go .eq. 'q' .or. &
  go .eq. 'Q') then
  goto 200
else
    write(*,3005)
3005    format ('Improper response.')
goto 6001
endif
200  end
!
!
subroutine r1
real*4 da, sl, pre, xlda, xlsl, xlp
integer*4 region, warn
common /c1/ da, sl, pre, xlda, xlsl, xlp, q72, q710, &
    b72, b710, u72, u710, warn
real*4 x(1,2),u2(2,2),xt(2,1),temp1(1,2),temp2(1,3),xtx(1,1), &
    u10(3,3),y(1,3),yt(3,1)
!
data u2 /0.02595, -3.07144E-7, &
    -3.07144E-7, 1.88380E-11 /
!
data u10 / 8.24294, -0.0036180, -6.47335, &
    -0.0036180, 2.53810E-6, 0.0027768, &
    -6.47335, 0.0027768, 5.09403 /
!
compute estimates
!
p= -1.1 + 0.00247*da + 0.0603*sl + 0.163*(pre-50)
prz=1-(exp(p)/(1+exp(p)))
q72=0.35 + 5.6E-5*da**2
q710=-7.1 + 0.0072*da + 5.5*sl**0.093
!
compute prediction limits
!
x(1,1)=1.0
x(1,2)=da
xt(1,1)=1.0
xt(2,1)=da
call multiply(temp1,x,u2,1,2,2,1,1,2)
call multiply(xtx,temp1,xt,1,2,1,1,1,2)
v=xtx(1,1)
s=sqrt(0.31427**2+v)
t=10**2*(1.78*s)
u72=q72*t
b72=q72/t
if (prz.gt.0.5) then
q72=0.0
b72=0.0
u72=0.0
end if
y(1,1)=1.0
y(1,2)=da
y(1,3)=sl
yt(1,1)=1.0
yt(2,1)=da
yt(3,1)=sl
call multiply(temp2,y,u10,1,3,1,1,3)
call multiply(xtx,temp2,yt,1,3,1,1,1,3)
v=xtx(1,1)
s=sqrt(0.325**2+v)
t=10**(1.80*s)
u710=q710*t
b710=q710/t
if (prz.gt.0.1) then
  q710=0.0
  b710=0.0
  u710=0.0
end if
return
end
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
!
q710=0.0015*da**1.11*sl**0.63*(pre-50)**1.17
!
! compute prediction limits
!
x(1,1)=1.0
x(1,2)=xlda
x(1,3)=xlp
xt(1,1)=1.0
xt(2,1)=xlda
xt(3,1)=xlp
call multiply(temp1,x,u2,1,3,1,1,3)
call multiply(xtx,temp1,xt,1,3,1,1,1,3)
v=xtx(1,1)
s=sqrt(0.31696**2+v)
t=10**(1.67*s)
u72=(q72/1.03)*t
b72=(q72/1.03)/t
y(1,1)=1.0
y(1,2)=xlda
y(1,3)=xlsl
y(1,4)=xlp
yt(1,1)=1.0
yt(2,1)=xlda
yt(3,1)=xlsl
yt(4,1)=xlp
call multiply(temp2,y,u10,1,4,1,1,1,4)
call multiply(xtx,temp2,yt,1,4,1,1,1,4)
v=xtx(1,1)
s=sqrt(0.31705**2+v)
t=10**(1.67*s)
u710=(q710/1.03)*t
b710=(q710/1.03)/t
return
end
!
!

subroutine r3
real*4 xlda,xlsl, xlp, da, sl, pre
integer*4 region, warn
common /c1/ da, sl, pre, xlda, xlsl, xlp, q72, q710, &
    b72, b710, u72, u710, warn
real*4 x(1,3),u2(3,3),xt(3,1),temp1(1,3),xtx(1,1),u10(3,3)
data u2 / 0.63652, -0.15047, 0.040292, 0.089652, &
    -0.15047, 0.040292, 0.089652, &
    -0.40850, 0.089652, 0.27601 /
data u10 /1.38662, -0.32715, -0.89094, &
   -0.32715, 0.08746, 0.19511, &
   -0.89094, 0.19511, 0.60271 /

! compute estimates
! q72=8E-4*da**1.3*sl**1.46
q710=1.6E-5*da**1.58*sl**2.31
!
! compute prediction limits
!
x(1,1)=1.0
x(1,2)=xlda
x(1,3)=xlsl
xt(1,1)=1.0
xt(2,1)=xlda
xt(3,1)=xlsl
call multiply(temp1,x,u2,1,3,3,1,1,3)
call multiply(xtx,temp1,xt,1,3,1,1,1,3)
v=xtx(1,1)
s=sqrt(0.21556**2+v)
t=10**(1.72*s)
u72=(q72/1.04)*t
b72=(q72/1.04)/t
call multiply(temp1,x,u10,1,3,3,1,1,3)
call multiply(xtx,temp1,xt,1,3,1,1,1,3)
v=xtx(1,1)
s=sqrt(0.33837**2+v)
t=10**(1.72*s)
u710=(q710/1.07)*t
b710=(q710/1.07)/t
return
end
subroutine multiply(prod,x,y,k1,k2,k3,n1,n2,n3)
integer*4 i,j,k1,k2,k3,n1,n2,n3
!
! matrix multiplication
!
real*4 prod(n1,k3), x(n2,k2), y(n3,k3), sum
do 3 i=1,k1
   do 2 k=1,k3
      sum=0.
      do 1 j=1,k2
         sum=sum+x(i,j)*y(j,k)
      1 continue
      prod(i,k)=sum
   2 continue
3 continue
return
end